



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

For the purpose of this problem, assume that f is continuous at $x=0$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.
First, show that $f(x) = cx$ for all $x \in \mathbb{Q}$. To do this, use the fact that f is continuous at $x=0$ and the fact that f is additive. Then, use the fact that \mathbb{Q} is dense in \mathbb{R} to show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

- (a) Show that $f(x) = cx$ for all $x \in \mathbb{Q}$.
- (b) Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume that f is continuous at $x=0$. Show that $f(x) = cx$ for all $x \in \mathbb{R}$.